fessor Boys. See paper "On the Production Properties and Some Suggested Uses of the Finest Fibres," *Phil. Mag.* 1887 June.

For silvering them I prefer Brashear's method (described at length in a paper by the writer, "On Silvering Solutions and Silvering," *Astrophysical Journal*, vol. 1, page 252, 1895 March). To get a good coat it is, however, necessary to clean the fibre very carefully. For this purpose I have found it best to use in succession very hot solutions of strong nitric acid and strong potash, and then wash in several changes of clean distilled water before immersing in the silvering solution. In order to handle the fibres during these operations I have found it convenient to mount them on a small rectangular glass frame with a bent handle, similar to that shown in the figure. The ends of the fibres are secured to the upper and lower bars of the frame by fused (not dissolved) shellac, which resists well the action of the hot acid and alkali. In lifting the frame in and out of the solutions care should be taken to move it vertically up and down, otherwise the fine fibres will be broken by the surface tension of the liquid surface. On such a frame a dozen fibres may be silvered at once, and then set aside for use as desired.

Another great advantage which such silvered fibres would be likely to possess when used in bright wire micrometers, *i.e.* that of perfect and uniform illumination, was pointed out by Professor Burnham in the course of a recent conversation with the writer on this subject. To get the best results in such a case a cylindrical mirror should be placed behind the wire (on the side away from the source of illumination) with its axis parallel to the length of the wire.

Yerkes Observatory:
1897 *May* 29.

Notes on the Reduction of Stellar Photographs.

By Prof. Arthur A. Rambaut, M.A., D.Sc.

All who have closely followed the recent developments in the use of photography as a means of measuring the positions of stars must have been impressed with the great value of the methods proposed and advocated by Professor Turner in the several papers which he has published on the subject.

From the point of view taken up by him—which is in the main that of a partaker in the International Photographic Survey of the Heavens, although many of his results have a wider application and are largely useful in facilitating operations and increasing accuracy whatever the object of research may be —from this point of view it is difficult to see in what way much further simplification in the reduction of the plates is to be introduced.

No method for determining the relative positions on overlapping plates can be more simple or symmetrical than that proposed by him and adopted, I understand, at Greenwich, Oxford, and other places.

The secret of this remarkable simplification lies in the fact of his adhering strictly to rectangular coordinates, and avoiding carefully as long as it is possible the use of the old and time-honoured system of right ascension and declination, which the fact of our inhabiting a rotating body obliges us in the last extremity to use in some form or other when we desire to define the position of a celestial body in space.

I expect, however, that there are many workers in this line, who, like myself, are occupied in taking photographs under conditions different from those considered by Professor Turner, M. Loewy, and others, in connection with the Astrophotographic Catalogue, and who find the methods advocated in the *Bulletin du Comité International Permanent pour l'Exécution Photographique de la Carte du Ciel* are not exactly suitable to their particular requirements.

I have been recently engaged in considering the best mode of reducing the plates taken at Dunsink with our 15-inch reflecting telescope, in which we have a means of adjusting the plate with a fair degree of precision, and of thus keeping it very approximately "squared on" to the axis of the telescope, but the actual R.A. and declination of the point to which the telescope is directed is not known with any considerable degree of accuracy, and the plate is merely inserted in an ordinary slide without any particular care as to its orientation.

For work such as we are engaged upon, in which the objects photographed are scattered in all parts of the sky, the principle of the "rattachment" of plates, recommended by Loewy, Turner, and others, for the Astrographic Catalogue, and the elaborate mode of reduction which the former has founded upon it are clearly inadmissible, and some more applicable method must be sought.

In a case of this sort if we have a number of reference stars distributed fairly symmetrically over the plate so that the lines joining them would form a polygon inside of which lie the object or objects whose position it is desired to ascertain, it is possible to attain an accuracy at least equal to the mean accuracy in the positions of the reference stars, and any inaccuracy introduced by small errors in the adopted positions of the latter will not affect the determination of parallax or proper motion provided that the same set of reference stars is used on each occasion.

In what follows I suppose that the measures do not extend over a space of more than a square degree. This is only one quarter of the space covered by one of the plates of the Astrophotographic Catalogue, but this limitation will enable us to dispense with many of the smaller terms which complicate the reduction, and which only become sensible when distances greater than 2000″ have to be considered, while giving us as

wide a region around the centre of each plate in which to find the reference stars as is recommended for the reduction of the Catalogue plates.

In a case such as I have suggested the advantages of using rectangular coordinates can, I think, be best attained by having recourse at once to a system of what Professor Turner calls "standard" coordinates. These are coordinates referred to the projections of the hour circle and of the great circle at right angles to it passing through the origin.

But under the conditions I have supposed we do not know the R.A. and declination of the origin from which our measures are made on the plate, and if we assume an origin for the standard coordinates we do not know precisely the point on the plate representing it.

We have therefore two systems of rectangular axes, one in a tangent plane to the sphere at a point whose R.A. and declination we assume, and the other, that to which the measures are made, whose position in the plane of the plate is known. And what we require to find is a means of passing from one of these systems to the other.

This is afforded us by a comparison of the coordinates of the reference stars with regard to each of them.

Now let us denote by O the point of the plate which we take as the zero of measurement, while the centre of the plate (C) is defined as the point in which the normal to it from the centre of the objective, or from the centre of curvature of the mirror, cuts the plate. If we take a pair of axes passing through the centre of the sphere parallel to the axes of measurement during the exposure, then these two with the direction of the normal to the plate through C form a system of three mutually rectangular axes. Let the coordinates of the image (S) of a star σ referred to there be x, y, z.

If now we take another system of rectangular axes passing through the centre of the sphere of which the axes of x and y are parallel to our assumed standard axes, and if these make angles α, β, γ; α', β', γ'; α'', β'', γ'' respectively with the previous system, and if the coordinates of the star σ referred to these be X, Y, Z we have

$$\mathrm{X}:\mathrm{Y}:\mathrm{Z}=x\cos\alpha+y\cos\beta+z\cos\gamma : x\cos\alpha'+y\cos\beta'+z\cos\gamma' : x\cos\alpha''+y\cos\beta''+z\cos\gamma''$$

Therefore

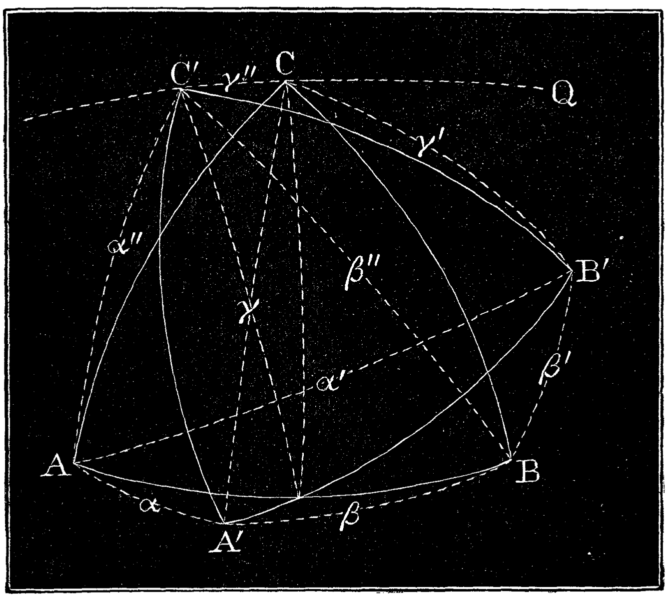
$$\left.\begin{aligned}\frac{\mathrm{X}}{\mathrm{Z}}&=\frac{\dfrac{x}{z}\dfrac{\cos\alpha}{\cos\gamma''}+\dfrac{y}{z}\dfrac{\cos\beta}{\cos\gamma''}+\dfrac{\cos\gamma}{\cos\gamma''}}{\dfrac{x}{z}\dfrac{\cos\alpha''}{\cos\gamma''}+\dfrac{y}{z}\dfrac{\cos\beta''}{\cos\gamma''}+1}\\ \text{and}\qquad\qquad&\\ \frac{\mathrm{Y}}{\mathrm{Z}}&=\frac{\dfrac{x}{z}\dfrac{\cos\alpha'}{\cos\gamma''}+\dfrac{y}{z}\dfrac{\cos\beta'}{\cos\gamma''}+\dfrac{\cos\gamma'}{\cos\gamma''}}{\dfrac{x}{z}\dfrac{\cos\alpha''}{\cos\gamma''}+\dfrac{y}{z}\dfrac{\cos\beta''}{\cos\gamma''}+1}\end{aligned}\right\} \quad \cdots\cdots \quad (1)$$

But $-Z$ is the radius of the sphere while $-z$ is the focal length of the telescope. Hence $-\frac{X}{Z}, -\frac{Y}{Z}$ are the standard coordinates (ξ and η) of the star expressed in radians, while $-\frac{x}{z}, -\frac{y}{z}$ are the coordinates of the star's image on the plate expressed in the same manner. Now we may put $-Z$ equal to unity, and if we denote $-\frac{1}{z}$ by λ (where λ is the value of a unit of the screw or scale with which the measures are made) we find

$$\left.\begin{aligned} \xi &= \frac{\lambda\,(ax+by)+c}{1-\lambda\,(kx+ly)} \\ \eta &= \frac{\lambda\,(dx+ey)+f}{1-\lambda\,(kx+ly)} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (7)$$

These are the expressions found by Professor Turner in his first paper on this subject (*Monthly Notices*, liv. 1) except that he supposes $\lambda = 1$, but this method of deducing them has the distinct advantage of showing plainly the geometrical meaning of each of the quantities involved.

For instance, from equations (1) we can exhibit in a very symmetrical manner the relations between the coefficients a, b, c, &c., deduced by Professor Turner in his paper in the *Monthly Notices*, vol. lvii. 3. For this purpose we have recourse to the accompanying figure in which A, B, C, A′, B′, C′ are the points in which the two sets of axes intersect the sphere.



Then

$$AA' = \alpha, \quad BA' = \beta, \quad CA' = \gamma$$

$$AB' = \alpha', \quad BB' = \beta', \quad CB' = \gamma'$$

and

$$AC' = \alpha'', \quad BC' = \beta'', \quad CC' = \gamma''$$

and since C and C′ are the poles of AB and A′B′ respectively, therefore the angle APA′$=\gamma''$. Also the angle QCA is Professor Turner's α which I denote by p, and QC′A′ is Professor Turner's β which I denote by q.

Then since P is the pole of CC′ it follows that

$$\text{AP}\,(=\text{ACP}) = p - \frac{\pi}{2}.\ \text{PB} = \pi - p$$

and

$$\text{A}'\text{P}\,(=\text{A}'\text{C}'\text{P}) = q - \frac{\pi}{2}.\ \text{PB}' = \pi - q$$

Hence it follows at once that

$$\begin{aligned}
\cos\alpha &= \sin p \sin q + \cos p \cos q \cos\gamma'' \\
\cos\beta &= -\cos p \sin q + \sin p \cos q \cos\gamma'' \\
\cos\gamma &= \cos q \sin\gamma'' \\
\cos\alpha' &= -\sin p \cos q + \cos p \sin q \cos\gamma'' \\
\cos\beta' &= \cos p \cos q + \sin p \sin q \cos\gamma'' \\
\cos\gamma' &= \sin q \sin\gamma'' \\
\cos\alpha'' &= -\cos p \sin\gamma'' \\
\cos\beta'' &= -\sin p \sin\gamma''
\end{aligned}$$

Hence,

$$\left.\begin{aligned}
a &= \frac{\cos\alpha}{\cos\gamma''} = \cos p \cos q + \sin p \sin q \sec\gamma'' \\
b &= \frac{\cos\beta}{\cos\gamma''} = \sin p \cos q - \cos p \sin q \sec\gamma'' \\
c &= -\frac{\cos\gamma}{\cos\gamma''} = -\cos q \tan\gamma'' \\
d &= \frac{\cos\alpha'}{\cos\gamma''} = \cos p \sin q - \sin p \cos q \sec\gamma'' \\
e &= \frac{\cos\beta'}{\cos\gamma''} = \sin p \sin q + \cos p \cos q \sec\gamma'' \\
f &= -\frac{\cos\gamma'}{\cos\gamma''} = -\sin q \tan\gamma'' \\
k &= \frac{\cos\alpha''}{\cos\gamma''} = -\cos p \tan\gamma'' \\
l &= \frac{\cos\beta''}{\cos\gamma''} = -\sin p \tan\gamma''
\end{aligned}\right\} \quad .\ .\ .\ .\ (3)$$

These are the relations found by Professor Turner. From the values of these quantities expressed in terms of the direction-cosines, we see that

$$a^2 + b^2 + c^2 = d^2 + e^2 + f^2 = 1 + k^2 + l^2 = \sec^2\gamma'',$$

relations which afford an important check on the accuracy of the computation.

From these equations, again, it appears that

$$c-k=(\cos p-\cos q)\tan\gamma''=-2\sin\frac{1}{2}(p-q)\sin\frac{1}{2}(p+q)\tan\gamma''$$

and

$$f-l=(\sin p-\sin q)\tan\gamma''=\quad 2\sin\frac{1}{2}(p-q)\cos\frac{1}{2}(p+q)\tan\gamma''$$

But $p-q$ can never exceed α, so that the differences $c-k$ and $f-l$ are of the order $\alpha\gamma''$, and hence to a first approximation may be neglected.

If we denote by x', y' the coordinates of the image S of a star σ, measured parallel to rectangular axes, passing through any point O on the plate, and if x_0, y_0 are the coordinates of the centre C referred to the same axes, then the coordinates of S referred to parallel axes through C are $x=x'-x_0$, $y=y'-y_0$.

For the origin of standard coordinates, we may take any point of the sphere whatever, the right ascension and declination of which we denote by A and D, and we shall have equations (2) connecting ξ, η, with x, y.

But if γ'' is a small quantity, *i.e.*, if the centre of the plate is close to the origin of standard coordinates—a condition which it is easy to fulfil—these expressions may be very much simplified.

For if we substitute from equations (3), neglecting powers of γ'' and λx, λy, beyond the third, and denoting the angle $g-p$ by ϕ, we find,

$$\xi=\frac{\lambda(x\cos\phi-y\sin\phi)+\cos q\,.\,\gamma''+\frac{1}{2}\sin q\,.\,\lambda(x\sin p-y\cos p)\gamma''^2+\frac{1}{3}\cos q\,.\,\gamma''^3}{1+\lambda(x\cos p+y\sin p)(\gamma''+\frac{1}{3}\gamma''^3)}.$$

Now, if γ'' does not exceed 100'', we may reject its cube and terms of the first order in x, y, when multiplied by its square, and of the second order multiplied by γ''.

To this order of approximation we find simply

$$\xi=\lambda(x\cos\phi-y\sin\phi)+\cos q\,.\,\gamma'',$$

while in a similar manner we obtain

$$\eta=\lambda(x\sin\phi+y\cos\phi)+\sin q\,.\,\gamma''.$$

Hence we see that $\cos q\,.\,\gamma''$ and $\sin q\,.\,\gamma''$ are the standard coordinates of the centre with regard to (A, D). Denoting them by ξ_0, η_0, we have finally,

$$\left.\begin{aligned}\xi-\xi_0&=\lambda\left[(x'-x_0)\cos\phi-(y'-y_0)\sin\phi\right]\\ \eta-\eta_0&=\lambda\left[(x'-x_0)\sin\phi+(y'-y_0)\cos\phi\right]\end{aligned}\right\}\quad\ldots\ldots\quad(4)$$

These are the ordinary formulæ of transformation from one pair of rectangular axes to another, and their applicability

depends on the fact that the assumption we have made as to the magnitude of γ'' is equivalent to assuming that the centre of the plate is so near the origin of standard coordinates, that the straight lines into which the latter project are sensibly at right angles to each other within the limits of the plate.

By means of the equations (4), we can, from the standard coordinates of two known stars, determine the values of λ and ϕ, whatever their magnitude may be, but in practice ϕ will always be a small angle, and λ will differ but little from unity. In this case assume

$$\lambda \cos \phi = 1 + \mu \cos \nu = 1 + \mathrm{P}$$
$$\lambda \sin \phi = \quad \mu \sin \nu = \mathrm{Q},$$

where P and Q are both small quantities. Then denoting by the subscript figures 1 and 2, the coordinates relating to the two reference stars, we have

$$(\xi_1 - \xi_2) - (x'_1 - x'_2) = \mu \cos \nu\, (x'_1 - x'_2) - \mu \sin \nu\, (y'_1 - y'_2)$$
$$(\eta_1 - \eta_2) - (y'_1 - y'_2) = \mu \sin \nu\, (x'_1 - x'_2) + \mu \cos \nu\, (y'_1 - y'_2).$$

Then assuming

$$\tan \psi_{12} = \frac{(\eta_1 - \eta_2) - (y'_1 - y'_2)}{(\xi_1 - \xi_2) - (x'_1 - x'_2)} \text{ and } \tan \theta_{12} = \frac{y'_1 - y'_2}{x'_1 - x'_2}$$

we have $\tan \psi_{12} = \tan (\nu + \theta_{12})$ or $\nu = \psi_{12} - \theta_{12}$.

Also

$$\sqrt{[(\xi_1 - \xi_2) - (x'_1 - x'_2)]^2 + [(\eta_1 - \eta_2) - (y'_1 - y'_2)]^2} = \mu \sqrt{(x'_1 - x'_2)^2 + (y'_1 - y'_2)^2},$$

whence

$$\mu = \frac{(\xi_1 - \xi_2) - (x'_1 - x'_2)}{x'_1 - x'_2} \cdot \frac{\cos \theta_{12}}{\cos \psi_{12}} = \frac{(\eta_1 - \eta_2) - (y'_1 - y'_2)}{y'_1 - y'_2} \frac{\sin \theta_{12}}{\sin \psi_{12}}.$$

Equations (4) may be written

$$\xi - x' - \mathrm{P}x' + \mathrm{Q}y' = \xi_0 - x_0 - \mathrm{P}x_0 + \mathrm{Q}y_0 = \bar{\xi}$$

and

$$\eta - y' - \mathrm{Q}x' - \mathrm{P}y' = \eta_0 - y_0 - \mathrm{Q}x_0 - \mathrm{P}y_0 = \bar{\eta}.$$

Substituting the values of P and Q in the left hand members, we find the values of $\bar{\xi}$, $\bar{\eta}$, which are the coordinates of O, referred to the standard axes through (A, D).

If the values of P and Q, as derived from different pairs of reference stars, are sensibly the same, so that the various values of $\bar{\xi}$ and $\bar{\eta}$ do not differ by more than the probable error in the coordinates of the stars will account for, we need go no further, but may assume P, Q, $\bar{\xi}$, $\bar{\eta}$ as the constants of the plate.

If, however, large differences occur in the values of P, Q, $\bar{\xi}$, $\bar{\eta}$, it follows that γ'' is not as small a quantity as is contemplated above, and it will be necessary to transform the standard axes to a point more nearly represented by the centre of the plate.

Of course, the quantities ξ, η, $\bar{\xi}$, $\bar{\eta}$ are *apparent* coordinates, and in deducing them from the known declinations and right ascension, the effects of refraction and aberration must be applied before they are compared with the measures.

The true values of these quantities are found from the formulæ

$$\left.\begin{aligned} \xi &= \frac{\cos\delta\sin(\alpha-A)}{\sin\delta\sin D+\cos\delta\cos D\cos(\alpha-A)} \\ \text{and}\qquad & \\ \eta &= \frac{\sin\delta\cos D-\cos\delta\sin D\cos(\alpha-A)}{\sin\delta\sin D+\cos\delta\cos D\cos(\alpha-A)} \end{aligned}\right\} \quad \ldots\ldots \quad (5)$$

α and δ being the right ascension and declination of the star (compare Ball and Rambaut "On the Relative Positions of 223 Stars in the Cluster χ Persei." *Transactions of the Royal Irish Academy*, vol. xxx. pt. 4, p. 235).

These are the same expressions as those given by Professor Turner in the *Monthly Notices*, vol. liv., 1, p. 13.

In most cases the approximate formulæ deduced from these will be found more convenient. These are, denoting $\alpha-A$ and $\delta-D$ by $\Delta\alpha$ and $\Delta\delta$ respectively,

$$\xi = \cos D \,.\, \Delta\alpha - \sin 1'' \,.\, \sin D \,.\, \Delta\alpha \,.\, \Delta\delta - \frac{1}{2}\sin^2 1'' \,.\, \cos D \,.\left(\frac{1}{3}-\cos^2 D\right).\, \Delta\alpha^3,$$

and

$$\eta = \Delta\delta + \frac{1}{4}\sin 1'' \,.\, \sin 2D \,.\, \Delta\alpha^2 + \frac{1}{3}\sin^2 1'' \,.\, \Delta\delta^3 + \frac{1}{2}\sin^2 1'' \,.\, \cos 2D \,.\, \Delta\alpha^2 \,.\, \Delta\delta.$$

For computing the coefficient of $\Delta\alpha^3$ in the first of these the following table for Log. $(\frac{1}{3}-\cos^2 D)$ is useful, three places of decimals being amply sufficient for any cases likely to occur in practice.

Table for Log $\left(\frac{1}{3} - \cos^2 D\right)$.

D.	Log $\left(\frac{1}{3}-\cos^2 D\right)$.		D.	Log $\left(\frac{1}{3}-\cos^2 D\right)$.		D.	Log $\left(\frac{1}{3}-\cos^2 D\right)$.		D.	Log $\left(\frac{1}{3}-\cos^2 D\right)$.	
0°	−9·824	0	23°	−9·711	11	45°	−9·222	48	68°	+9·286	25
1	·824	1	24	·700	12	46	·174	54	69	·311	24
2	·823	1	25	·688	12	47	·120	62	70	·335	22
3	·822	1	26	·676	13	48	−9·058	71	71	·357	19
4	·821	2	27	·663	14	49	−8·987	85	72	·376	18
5	·819	2	28	·649	14	50	·902	105	73	·394	16
6	·817	3	29	·635	15	51	·797	137	74	·410	15
7	·814	3	30	·620	17	52	·660	201	75	·425	14
8	·811	3	31	·603	17	53	·459	376	76	·439	12
9	·808	4	32	·586	18	54	−8·083		77	·451	11
10	·804	5	33	·568	19	55	+7·633	681	78	·462	11
11	·799	4	34	·549	20	56	8·314	251	79	·473	9
12	·795	5	35	·529	22	57	·565	155	80	·482	8
13	·790	6	36	·507	23	58	·720	113	81	·490	7
14	·784	6	37	·484	25	59	·833	88	82	·497	6
15	·778	7	38	·459	27	60	·921	72	83	·503	5
16	·771	7	39	·432	28	61	8·993	60	84	·508	5
17	·764	7	40	·404	31	62	9·053	51	85	·513	4
18	·757	8	41	·373	33	63	·104	46	86	·517	2
19	·749	9	42	·340	36	64	·150	39	87	·519	2
20	·740	9	43	·304	39	65	·189	36	88	·521	1
21	·731	10	44	·265	43	66	·225	32	89	·522	1
22	·721	10	45	·222		67	·257	29	90	·523	
23	·711					68	·286				

The corresponding expressions for deducing $\Delta\alpha$ and $\Delta\delta$ from the values of ξ and η, are given in the above mentioned memoir by Ball and Rambaut, and are

$$\left.\begin{array}{l}\Delta\alpha \cos D = \xi + \sin 1'' \tan D \,.\, \xi\eta - \frac{1}{3}\sin^2 1'' \sec^2 D \,.\, \xi^3 + \sin^2 1'' \tan^2 D \,.\, \xi\eta^2 \\ \text{and} \\ \Delta\delta = \eta - \frac{1}{2}\sin 1'' \tan D \,.\, \xi^2 - \frac{1}{2}\sin^2 1'' \sec^2 D \,.\, \xi^2\eta - \frac{1}{3}\sin^2 1'' \,.\, \eta^3.\end{array}\right\} (6)$$

These formulæ contain all the terms which it is necessary to consider, provided that ξ and η do not exceed 2000″, and that D is less than 75°. This will include the vast majority of cases likely to occur in practice, but if it be desired to extend the measures much beyond half a degree from the centre and for

regions nearer the pole, higher terms must be considered or the exact formulæ employed.

The terms of the fourth order are given in a paper by Östen Bergstand, "Sur la Réduction des Mesures Micrométriques des Clichés Photographiques Stellaires" (*Öfversigt af Kongl. Vetenskaps Akademiens Förhandlingar*, 1896, No. 7), while those of the fourth and fifth orders will be found in a paper by Dr. Jacoby, on "The Reduction of Stellar Photographs," contained in the *Contributions from the Observatory of Columbia University, New York*, No. 10.

In the latter paper the author prefers to employ the quantity $\frac{1}{15}\xi$ sec D instead of ξ, which renders his formulæ somewhat cumbersome and more unattractive than is necessary, but they can be easily transformed so as to express $\Delta\alpha$ and $\Delta\delta$ in terms of ξ and η directly.

Refraction.

In the majority of cases likely to occur in practice the formulæ for the refraction, deduced by Professor Turner in his paper published in the *Monthly Notices*, vol. liv. No. 1, containing only terms of the first order in ξ and η, will be amply sufficient. For the limits at which the terms of the second order and those depending on $\frac{d\,\beta_0}{d\,\zeta}$ and $\beta_0{}^2$ become appreciable, compare a paper by the present author entitled "The Corrections for Refraction to Measures of Stellar Photographs," in the *Astronomische Nachrichten*, No. 3125.*

Professor Turner's formulæ are

$$d\xi = -\beta_0\,(1+\mathrm{X}^2)\,\xi - \beta_0\mathrm{XY}\,.\,\eta$$

and

$$d\eta = -\beta_0\mathrm{XY}\,.\,\xi - \beta_0\,(1+\mathrm{Y}^2)\,.\,\eta$$

in which X, Y denote the coordinates of the projection of the zenith on the plane containing the standard axis, and β_0 is the coefficient of refraction, corresponding to the point where these standard axes intersect at the middle of the exposure.

The simplicity and symmetry of these expressions must recommend them to every one who is familiar with the more cumbrous formulæ, which become necessary when the relative positions of stars are defined by means of their differences of R.A. and declination.

But although, as they stand, they are all that could be desired from an analytical point of view, yet as Professor Turner leaves them they are not as convenient as they might be for numerical computation.

* See also Kapteyn, *Bulletin du Comité International Permanent*, etc., Tome III., and Bergstrand, *Öfversigt af Kongl. Vetenskaps Akademiens Förhandlingar*, 1897, No. 2.

They may, however, be very readily transformed into a more convenient form. For since in the case of the zenith—for which $\alpha = \theta$, the sidereal time at the middle of the exposure, and $\delta = \phi$—we have, see equations (5).

$$X = \frac{\cos\phi \sin(\theta - A)}{\sin\phi \sin D + \cos\phi \cos D \cos(\theta - A)}$$

and

$$Y = \frac{\sin\phi \cos D - \cos\phi \sin D \cos(\theta - A)}{\sin\phi \sin D + \cos\phi \cos D \cos(\theta - A)}$$

if we take m and n such that

$$\cos n = \cos\phi \sin(\theta - A)$$

$$\sin n \sin m = \cos\phi \cos(\theta - A)$$

and

$$\sin n \cos m = \sin\phi$$

we find

$$X = \frac{\cos n}{\sin n \sin(m + D)} \text{ and } Y = \frac{\sin n \cos(m + D)}{\sin n \sin(m + D)}.$$

Hence,

$$1 + X^2 = \frac{1 - \sin^2 n \cos^2(m + D)}{\sin^2 n \sin^2(m + D)}$$

$$XY = \frac{\sin n \cos n \cos(m + D)}{\sin^2 n \sin^2(m + D)}$$

$$1 + Y^2 = \frac{\sin^2 n}{\sin^2 n \sin^2(m + D)}$$

But if ζ denote the zenith distance

$$\cos\zeta = \sin n \sin(m + D),$$

—a quantity we have to compute in order to obtain β_0—and if we take

$$\sin n \cos(m + D) = \cos n'$$

we have

$$1 + X^2 = \frac{\sin^2 n'}{\cos^2\zeta}, \quad XY = \frac{\cos n' \cos n}{\cos^2\zeta}, \quad 1 + Y^2 = \frac{\sin^2 n}{\cos^2\zeta}.$$

But m and n, given by the equations

$$\tan m = \cot\phi \cos(\theta - A)$$

and

$$\cot n = \sin m \tan(\theta - A),$$

are the quantities ordinarily used for computing refraction and parallactic angle and are taken directly, with the hour angle as argument, from tables which should be computed for each observatory.

We have then merely to compute

$$\cos \zeta = \sin n \sin (m + D),$$

whence we find ζ, and therefore β_0, and

$$\cos n' = \sin n \cos (m + D)$$

whence we find $\sin n'$ and the refraction formulæ as applied to standard rectangular coordinates become simply

$$d\xi = -\frac{\beta_0}{\cos^2 \zeta}[\sin^2 n' \,.\, \xi + \cos n' \cos n \,.\, \eta]$$

and

$$d\eta = -\frac{\beta_0}{\cos^2 \zeta}[\cos n' \cos n \,.\, \xi + \sin^2 n \,.\, \eta],$$

while equations of the same form, but with a different value of β_0, will give the correction to the apparent ξ and η as deduced from the measures to reduce them to true standard coordinates before transforming into $\Delta\alpha$ and $\Delta\delta$ by means of equations (6).

Aberration.

In the paper referred to above, Professor Turner finds for the effect of aberration the still simpler formulæ

$$d\xi = -\frac{\gamma_0}{\sqrt{1 + X^2 + Y^2}} \,.\, \xi \qquad \text{and} \qquad d\eta = -\frac{\gamma_0}{\sqrt{1 + X^2 + Y^2}} \,.\, \eta$$

in which γ_0 is the constant of aberration, and (X, Y) is the projection of the "Earth's Way" on the plane of the plate.

As in the case of the refraction formulæ, these expressions may be readily transformed so as to be more convenient for numerical computation.

By the use of Bessel's Day Numbers from the Ephemeris combined with the "Tables for facilitating the Computation of Star Constants," compiled by the late Dr. Stone, the computation of this correction assumes an exceedingly simple form.

For we have

$$\frac{1}{\sqrt{1 + X^2 + Y^2}} = \cos \theta_0$$

where θ_0 denotes the angle between the "Earth's Way" and the point of the sky represented by the centre of the plate.

Hence,

$$\frac{-\gamma_0}{\sqrt{1 + X^2 + Y^2}} = -\gamma_0 \cos \theta_0 = -\gamma_0[\sin \bar{\delta} \sin D + \cos \bar{\delta} \cos D \cos (\bar{a} - A)]$$

in which $\bar{a}$ and $\bar{\delta}$ denote the R.A. and declination of the "Earth's

Way." But, $\odot$ denoting the Sun's longitude and ϵ the obliquity of the ecliptic, we have

$$\cos \bar{a} \cos \bar{\delta} = \sin \odot$$
$$\sin \bar{a} \cos \bar{\delta} = -\cos \odot \cos \epsilon$$
$$\sin \bar{\delta} = -\cos \odot \sin \epsilon$$

Hence,

$$-\gamma_0 \cos \theta_0 = \gamma_0[\cos \odot \sin \epsilon \sin \mathrm{D} - \sin \odot \cos \mathrm{D} \cos \mathrm{A} + \cos \odot \cos \epsilon \cos \mathrm{D} \sin \mathrm{A}].$$

Writing D′ for $\frac{\pi}{2} - \mathrm{D}$ this becomes

$$-\gamma_0 \cos \theta_0 = -\gamma_0[\cos \odot \cos \epsilon\{-\cos \mathrm{D}' \tan \epsilon - \sin \mathrm{D}' \sin \mathrm{A}\} + \sin \odot (-\sin \mathrm{D}' \cos \mathrm{A})]$$

But

$$\left.\begin{array}{l} -\gamma_0 \cos \odot \cos \epsilon = \mathrm{A} \\ -\gamma_0 \sin \odot \qquad = \mathrm{B} \end{array}\right\} \text{from the Ephemeris}$$

And—without having regard to signs—

$$\sin \mathrm{D}' \sin \mathrm{A} = a^1, \ \cos \mathrm{D}' \tan \epsilon = sa^1, \text{ and } \sin \mathrm{D}' \cos \mathrm{A} = b^1,$$

a^1, sa^1, b^1 being the quantities denoted by the same symbols in Dr. Stone's tables corresponding to the declination D′.

We thus find that

$$d\xi = \{\mathrm{A}(a^1 + sa^1) + \mathrm{B}.b^1\} \sin 1''. \quad \xi$$

with a similar expression for $d\eta$.

With regard to the signs of a^1, sa^1, and b^1, it is easy to see that, adopting the convention that ξ increases in the direction of increasing R.A. and η in that of increasing declination, they may be taken from the following table:—

Signs of the Constants.

North of Equator.			R.A.	South of Equator.		
a^1.	sa^1.	b^1.	h h	a^1.	sa^1.	b^1.
−	−	+	0 to 6	−	+	+
−	−	−	6 ,, 12	−	+	−
+	−	−	12 ,, 18	+	+	−
+	−	+	18 ,, 24	+	+	+

It is not necessary to consider the corrections for Precession and Nutation. For since these are merely changes affecting the positions of the circles from which R.A. and Declination are measured, and have no effect on the relative positions of the stars themselves, they need not be taken into account until we convert the measured rectangular coordinates into differences of R.A. and declination, and then the corrections can best be computed by means of the ordinary tables.

The only remaining disturbance affecting the position of a star which it is necessary to consider is the annual displacement due to

Parallax.

This may be at once deduced from the formulæ for the aberration given by Professor Turner in note iii of his paper in the *Monthly Notices*, vol. liv. No. 1. If we write π for γ_0 (π being the annual parallax), and let X, Y be the coordinates of the projection of the Sun on the plate, then the parallactic displacement is

$$d\xi = \pi \frac{X-\xi}{\sqrt{1+X^2+Y^2}}$$

and

$$d\eta = \pi \frac{Y-\eta}{\sqrt{1+X^2+Y^2}}$$

in which the eccentricity of the Earth's orbit is neglected.

In these expressions we may reject ξ and η and we get as the effect of parallax

$$d\xi = \frac{\pi X}{\sqrt{1+X^2+Y^2}}$$

and

$$d\eta = \frac{\pi Y}{\sqrt{1+X^2+Y^2}}$$

Now

$$\frac{1}{\sqrt{1+X^2+Y^2}} = \cos \theta_0$$

θ_0 being in this case the angle between the direction of the Sun and that of the centre of the plate. Hence, α and δ being the Sun's R A. and declination

$$\frac{1}{\sqrt{1+X^2+Y^2}} = \sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)$$

But

$$X = \frac{\cos \delta \sin (\alpha - A)}{\sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)}$$

and

$$Y = \frac{\sin \delta \cos D - \cos \delta \sin D \cos (\alpha - A)}{\sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)}$$

Hence we have simply

$$d\xi = \pi \cos \delta \sin (\alpha - A)$$

and

$$d\eta = \pi (\sin \delta \cos D + \cos \delta \sin D \cos [\alpha - A])$$

or, substituting for α and δ in terms of the Sun's longitude and the obliquity of the ecliptic,

$$d\zeta = -\pi(\cos \odot \sin A - \sin \odot \cos \epsilon \cos A)$$

and

$$d\eta = -\pi(\cos \epsilon \sin A \sin D - \sin \epsilon \cos D) \sin \odot - \pi \cos \odot \sin D \cos A$$

in which $d\eta$ is the well known expression for the annual parallax in declination, while $d\xi$ is the parallax in R.A. multiplied by cos D.

Dunsink Observatory, Co. Dublin:
1897 *May* 28.

Photographic Observations of Comet b 1896.
By Professor Arthur A. Rambaut, M.A., Sc.D.

On the nights of 1896 April 29 and 30, May 8 and 11, photographs were taken of Comet *b* 1896 with the 15-inch reflecting telescope of the Dunsink Observatory. After that date, owing to increasing twilight and diminishing brightness of the comet, it could not be found in the 5-inch guiding telescope.

An exposure of ten minutes was given on each occasion, and except in the case of plate *c* on April 30 the movement was adjusted to sidereal rate, and the image of the comet accordingly appears as an elongated patch with ill-defined edges.

The middle point of the axis of this patch was taken as the position of the comet corresponding to the mean time of exposure.

The measures were made in the Troughton and Simms microscope, described in *Transactions of the Royal Irish Academy*, vol. xxx. part iv.

The microscope is provided with two screws, very approximately at right angles to each other, and readings were taken on both screws simultaneously.

The plate was then turned through exactly 90° by the position circle, and another set of measures taken. We thus obtained the rectangular coordinates independently on each screw, and these were separately reduced by the formulæ given in the preceding paper, the mean of the two being taken as the position of the comet given in the table below.

There is not the material for determining the probable error of these positions in the usual way, but an estimate of their precision may be gathered from the following results.

From the measures of +51°, 686 on plate *b*, which were